One, Two, Buckle My Buskin

by: R.L. Fowler

In the last issue of Labyrinth, G.W. Bennett ("Greek Mathematics: Part One") remarked how strange it was that the Greeks never bothered to invent an easy-to-use system of notation for numbers, even though they must have felt the lack greatly. The Romans, too, although they never showed the same interest in higher mathematics, could have made good use of a simpler system like our Arabic numerals in keeping the accounts of the imperial Treasury. However, as in most things, a lot depends on what you're used to.

The Greeks used the letters of their alphabet, with a few additions, to designate numbers. This was their system:

1-9: α, β, γ, δ, ε, ϛ (stigma), ζ, η, θ

10-90: ι, κ, λ, μ, ν, ξ, ο, π, ϙ (koppa)

100-900: ρ, σ, τ, υ, φ, χ, ψ, ω, ϡ (sampi)

The three signs stigma, koppa and sampi have been added to the standard Greek alphabet to make 27 signs available. 16 would be represented by ιϛ; 95 by ϙε; 674 by χοδ. From 1,000 on one used α through θ again, only with a distinguishing mark: ͵α = 1,000, ͵β = 2,000 and so on. Ten thousands ("myriads" in Greek) could be represented by writing M with a number above it, e.g. $\overset{\beta}{M}$ = 20,000; or you could write a letter with two dots over it, $\ddot{\beta}$. This gets us up to quite high numbers, beyond which we need not go here, although Greek mathematicians devised special systems which permitted them to go much farther still. Roman numerals you are of course familiar with: I, V, X, L, C, D, M; perhaps you aren't aware of their sign ∞ (also = 1,000), or their system of putting a single bar over a letter to indicate multiplication by 1,000 ($\overline{V}$ = 5,000) or of putting a sort of box over the number to indicate a factor of 100,000 ($\boxed{X}$ = 1,000,000).

Most simple calculations needed in everyday life could be done by the average person on his or her fingers. In Aristophanes' Wasps, for example, Philokleon is told not to bother getting his pebbles out, but to use his fingers instead to do a certain calculation. In question is a sum of 2000, a multiplication of 6 x 3,000 x 300 and a division of 2000 by 150. Part of the humour may lie precisely in old Philokleon's inability to do such reckoning (one imagines him furiously flapping his digits), but there is no doubt that one can count quite proficiently using this method.

When Bdelykleon told his father to forget his pebbles he was referring to a ready-made calculation device frequently used by the ancients. It's a sort of open-ended abacus (which machine they also had, by the way). They would draw columns in the dust or in sand spread on a table, then use pebbles as counters. The right hand column was units; the next one was tens; then hundreds, and so on. What's this, you say, the decimal system? Isn't that Arabic? No, it's very easy to come by and natural to use; after all, we do have ten fingers. By putting the appropriate number of pebbles in each column you can represent any number; to add, you represent your second number in the same way, then do some shuffling (if you end up with thirteen pebbles in the first column, for example, take away ten and add one in the second column). Subtraction works similarly, only the other way around; now and again you have to put in some extra pebbles in the lower column. If you were somewhat more practised, you could dispense with pebbles and arrange actual numbers in the columns. Suppose you wanted to add 1,424 + 103 + 12,281 = 13808. This is how it would look.

	͵α	υ	κ	δ
		ρ		γ
α̈	͵β	σ	π	α
α̈	͵γ	η		η

∞	CCCC	XX	IIII
	C		III
$\overline{XII}$	CC	LXXX	I
$\overline{XIII}$	DCCC		VIII

Note that one simply leaves a blank for 0; neither Greeks nor Romans had a sign for it.

Multiplication worked by breaking up the problem into a bunch of smaller problems that could be added together. For example, if you wanted to multiply 781 by 63, you did the following separate multiplications: 700 x 60; 700 x 3; 80 x 60; 80 x 3; 1 x 60; 1 x 3. Each of these is manageable in your head provided you know the basic multiplication tables. Using trusty abacus or handy corner of vacant lot, put down the individual results as they emerge, and add them up to 49,203. Division was somewhat more laborious, although it proceeded essentially along the same lines as now. If you want to divide 49,203 by 781, for example, you first divide 49,000 by 700 = 70; but since you know you've got an extra 81 to reckon with you reduce this and put down 6 in the tens column as the first number of your quotient. Multiply 781 by 60 using the method described above, subtract the result from 49,203 using your abacus, and press on to discover the second number of the quotient.

You can see that the clumsy system of notation does not really prevent ordinary calculations; it merely makes them slower. When it comes to doing exotic fractions or finding square roots, however, the difficulties can become formidable. One wonders if the authors of Greek mathematical treatises were not at least as interested in exquisite mental torture as they were in solving problems; the incredible circumlocutions necessary to discuss some of their subjects would daunt all but the most determined of us. Most people then and now never venture into this rarified atmosphere (fortunately). Fractions were only needed to deal with the subdivisions of coins and weights (see for example Horace's schoolboy in the _Ars Poetica_ 325 ff.). At the other extreme, large numbers tended to melt together into a vague "myriads" or "milia". Modern historians learn very early on in their training not to trust large figures given by an ancient source unless it can be shown how they were arrived at. Even then one occasionally wonders. Herodotus gives what appears to be the most careful calculation for the size of the Persian expeditionary force of 480 B.C. He begins with the size of the infantry: 1,700,000 men. The Persians had figured this much out for themselves; the way they did it shows that they, too, were not at home with large numbers. They first counted out 10,000 men (one myriad), then packed them all together as closely as they could. Next they drew a line around them all, and erected a low fence on this line. The first lot were marched out, and others were marched into the enclosure, close-ranked again. When there was no more room, the reasoning went, they had another 10,000 men. And so on until all were accounted for! Herodotus then reckoned the number of men in the navy by counting numbers and sizes of ships; finally he reasoned that the number of servants and other hangers-on would be roughly equal to the number of fighting men (not an unlikely assumption). The whole process looks to be as scientific as one could expect; yet the grand total arrived at is an incredible φκ̈η͵γτκ men! (5,283,320 to us.) It is inconceivable that so large a force marched and sailed across the north Aegean into Greece. Somewhere along the line, a few soldiers must have got themselves counted twice - maybe in an attempt to claim extra rations!